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Tunneling into the Energy Bands of the Intermediate State in Type-I Superconductors

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A tunneling experiment is proposed to verify the recently predicted band structure in the excitation spectrum of the laminar intermediate state. The theory shows that the energy bands are reflected by rapid oscillations of the differential tunneling conductance for small voltages of the order of the maximum value of the pair potential. In weak magnetic fields ($H \approx 0.1 H_c$) which produce normal regions of some 10^{-2} cm, the distance between two oscillation peaks is of the order of 10^{-5} to 10^{-6} V.

I. INTRODUCTION

Recently van Gelder calculated the excitation spectrum of a superconductor where the pair potential $\Delta(\vec{r})$ is a one-dimensional periodic step function[1] (Kronig model). He found that the eigenstates of the quasiparticle excitations are arranged in energy bands as is to be expected from the periodicity of the pair potential. Applying the Wentzel-Kramers-Brillouin-Jeffreys (WKBJ) method of solving the Bogoliubov equations of Bardeen *et al.*[2] to the laminar intermediate state with a periodic, but otherwise arbitrary, pair potential, the author obtained the general eigenvalue equations for the excitation energies with two variational parameters.[3] These equations reduce to van Gelder's result in the limit of the Kronig model.

Although the existence of energy bands in the intermediate state is well established theoretically, one may wonder if they can be verified experimentally. Owing to the shallowness of the pair potential wells and because of the macroscopic size of the normal and superconducting regions, a typical periodicity interval has a length of about 10^{-1} cm,[4,5] the bandwidths as calculated in Sec. II are very small. A forbidden band measures some 10^{-6} eV for weak magnetic fields. Therefore, the different band sequences for different values of quasiparticle momenta $\vec{k}_{\parallel}$ parallel to the phase boundaries fill in each other's gaps, and the band structure will be detected only by an experiment which singles out a well-defined band sequence. As has been suggested,[3] such an experiment may be tunneling into the intermediate state through an insulating barrier parallel to the normal superconducting phase boundaries (see Fig. 1). The tunneling probability is appreciable only for electrons with Fermi momentum k_F perpendicular to the barrier which face an allowed energy band with $\vec{k}_{\parallel} = 0$ in the superconductor. Because of the narrowness of the low-lying energy bands, the differential tunneling conductance calculated in Sec. III oscillates rapidly as a function of the applied bias voltage. In Sec. IV we discuss some of the experimental aspects to be considered in a measurement of the effect.

II. ENERGY BANDS

Since neither the existence of the energy bands nor the order of magnitude of their width depend upon the detailed form of the periodic pair potential, we may choose the simple Kronig model for a discussion of tunneling. In this model the eigenvalue equations[1,3] are in reduced units for energies, measured from the Fermi surface μ,

$E/\Delta \equiv \epsilon < 1$:

$$\cos[(k_\perp - k_F \cos\theta)2D] = \cos(a\epsilon/\cos\theta) \times \cosh[b(1-\epsilon^2)^{1/2}/\cos\theta] - [\epsilon/(1-\epsilon^2)^{1/2}]\sin(a\epsilon/\cos\theta) \times \sinh[b(1-\epsilon^2)^{1/2}/\cos\theta]; \quad (2.1a)$$

$E/\Delta \equiv \epsilon > 1$:

$$\cos[(k_\perp - k_F \cos\theta)2D] = \cos[a\epsilon/\cos\theta + b(\epsilon^2-1)^{1/2}/\cos\theta] - [\epsilon/(\epsilon^2-1)^{1/2} - 1]\sin(a\epsilon/\cos\theta) \times \sinh[b(\epsilon^2-1)^{1/2}/\cos\theta]\,. \quad (2.1b)$$

Equation (2.1b) can be obtained from Eq. (2.1a) by

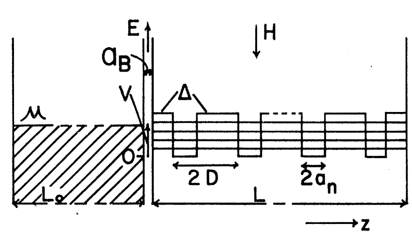

FIG. 1. Tunneling junction between a normal metal and a superconductor in the intermediate state. A few of the very narrow energy bands for $E < \Delta$ are indicated as energy levels.

changing $\epsilon < 1$ into $\epsilon > 1$. The symbols used in (2.1) have the following meaning: Δ is the maximum value of the pair potential in the superconducting regions; $k_\perp$ is the component normal to the phase boundaries of the propagation vector of the quasiparticle Bloch wave

$$\psi_{\vec{k}}(\vec{r}) = \chi_{\vec{k}}(z)\, e^{ik_\perp z} e^{i\vec{k}_\parallel(\vec{x}+\vec{y})}, \quad \chi_{\vec{k}}(z+2D) = \chi_{\vec{k}}(z)\,; \qquad (2.2)$$

$2D$ is the periodicity of the laminar intermediate-state structure (see Fig. 1); $\cos\theta$ is defined by $k_F^2\cos^2\theta/2m \equiv \mu - k_\parallel^2/2m$; thus, for quasiparticles at the Fermi surface, θ is the angle of incidence on the phase boundary measured against the z axis; $a \equiv (2m/k_F)\Delta DH/H_c = (2D/\pi\xi)H/H_c$ measures the width of the normal regions $2a_N = 2DH/H_c$ relative to the coherence length $\xi \equiv k_F/m\pi\Delta$; $\hbar = 1$; $b \equiv (2m/k_F)\Delta D(1 - H/H_c) = (2D/\pi\xi)(1 - H/H_c)$ measures the width $2(D - a_N)$ of the superconducting regions; and H_c is the critical magnetic field.

In the case $\epsilon \gg 1$, or if $H \geq H_c$ so that $b = 0$, Eqs. (2.1) give

$$(k_\perp - k_F\cos\theta)2D + 2n\pi = 2mDE_n/k_F\cos\theta,$$

$$n = \text{any integer}$$

$$E_n = (k_F/m)\cos\theta\,[(k_\perp + 2n\pi/2D) - k_F\cos\theta]\ . \qquad (2.3)$$

For n such that $k_\perp + 2n\pi/2D$ is the z component of a wave vector not far from the Fermi surface so that

$$\Delta k \equiv |k_\perp + 2n\pi/2D - k_F\cos\theta| \ll k_F\ ,$$

$$E_n = \Delta k k_F\cos\theta/m \approx k_\parallel^2/2m + (k_F\cos\theta + \Delta k)^2/2m - \mu$$

is the continuous normal quasiparticle spectrum for $E_n \ll \mu$. We can count the number of states belonging to different values of $k_\perp$ by applying periodic boundary conditions at the limits of the sample to the Bloch wave (2.2). If the sample length in z direction is L, then all

$$k_\perp = p2\pi/L, \quad p = \text{integer} \qquad (2.4)$$

are the discrete-allowed values of $k_\perp$. As one knows from solid-state band theory, in the reduced zone scheme, the maximum value of $k_\perp$ at which occurs Bragg reflection in the periodic structure is 2π divided by the periodicity interval. Thus,

$$(k_\perp)_{\max} = 2\pi/2D\ ,$$

and in each band there are $M = L/2D$ different states; $1 \leq p \leq M$; $L = 2D$ if $H \geq H_c$. The index n labels the bands which in the case of $E \gg \Delta$, $H < H_c$, according to Eq. (2.3), join each other smoothly as p varies between 1 and M.

Because of the macroscopic size of the normal and superconducting layers the constants a and b are rather large numbers. From the theory of Landau and Lifshitz one obtains[4] the lengths $2D \approx 6\times10^{-2}$ cm and $2a_N \approx 4\times10^{-2}$ cm for $H/H_c = 0.7$. Powder pattern pictures of Al for $H = 0.08H_c$ give $2D \approx 3\times10^{-1}$ cm (this number is a crude estimate of the author taken from Faber's pictures[6] as quoted by Livingston and Desorbo[5]) and $2a_N \approx 2.4\times10^{-2}$ cm; in that case $a = 80$ and $b = 900$. As long as H is not very close to H_c, b will always be a large number. Consequently, to a very good approximation we may write Eq. (2.1a) for $E < \Delta$ as

$$\cos[(k_\perp - k_F\cos\theta)2D]$$

$$= \frac{e^{b(1-\epsilon^2)^{1/2}/\cos\theta}}{(1-\epsilon^2)^{1/2}} \sin\left(\eta(\epsilon) - \frac{a\epsilon}{\cos\theta}\right), \qquad (2.5)$$

where

$$\eta(\epsilon) \equiv \arccos\epsilon\ .$$

Because of the extremely large amplitude $e^{b(1-\epsilon^2)^{1/2}/\cos\theta}$, Eq. (2.5) can be satisfied only in a very narrow range of energies ϵ around the values which make

$$\sin[\eta(\epsilon) - a\epsilon/\cos\theta] = 0\ .$$

Thus, for all practical purposes we may consider the energy bands for $E < \Delta$ as sharp levels satisfying the eigenvalue equation

$$\arccos\epsilon - a\epsilon/\cos\theta = n\pi\ . \qquad (2.6)$$

Equation (2.6) is exactly the same eigenvalue equation as for the energy spectrum of an isolated normal layer in an infinite superconductor[3,7]; approximate solutions of it are given in Eq. (3.16). The only difference is the M-fold degeneracy corresponding to the M possible $k_\perp$ values in each band.

For energies $E \gtrsim \Delta$ the second term on the right-hand side of Eq. (2.1b) produces the band gaps. Its amplitude $\epsilon/(\epsilon^2-1)^{1/2} - 1$ decreases hyperbole-like from 6.16 for $\epsilon = 1.01$ over 0.16 for $\epsilon = 2$ to 0 for large ϵ. Thus, the continuous spectrum (2.3) should begin for energies of about the order of 2Δ, whereas the lowest states between Δ and 2Δ still form very narrow bands due to the rapid oscillations of the right-hand side of Eq. (2.1b).

III. TUNNELING

The tunneling probability per unit time w for a net current flow from the left side of the insulating layer into the intermediate state on the right (see Fig. 1) is given by[8,9]

$$w = \frac{\pi}{2LL_0 m^2} \sum_{k_z,\vec{k}_\parallel} \sum_{k'_z,\vec{k}'_\parallel} k_z k'_z e^{-2\kappa a_B} \delta_{\vec{k}_\parallel,\vec{k}'_\parallel}$$

$$\times C(d,E')\ \delta(E(\vec{k}) - E'(\vec{k}'))$$

$$\times [f(E(\vec{k}) - V) - f(E'(\vec{k}'))]\ , \quad (3.1)$$

$\vec{k} = \vec{k}_z + \vec{k}_\parallel$, and $\vec{k}' = \vec{k}'_z + \vec{k}'_\parallel$ are the quasiparticle wave vectors on the left and the right of the tunneling barrier of thickness a_B; E and E' are the corresponding energies; k_z' is the value of $k_\perp$ of Eq. (2.4) in the extended zone scheme; $\kappa \equiv [2m(U_0 - V) - k_z'^2]^{1/2}$, U_0 being the average height of the potential in the barrier and V the applied bias voltage times electron charge e; and $f(E)$ is the Fermi distribution function. The factor $C(d, E')$ has been introduced in order to take into account the reduction of the amplitude of the tunneling wave function by the decrease of the amplitude of the periodic part $\chi_{\vec{k}}(z)$ of the quasiparticle wave function (2.2) outside the normal regions[3] for $E < \Delta$; d is the distance from the tunneling barrier to the nearest normal region.

After summing over $\vec{k}_\parallel'$ we can change the sum over k_z' into an integral of the density $\rho_{r\perp}$ of those states on the right in which the particles move normal to the phase boundaries:

$$\sum_{k_z'} \to \int dE' \rho_{r\perp}(E') \ .$$

The sum over k_z, $\vec{k}_\parallel$ becomes

$$\sum_{k_z, \vec{k}_\parallel} \to \frac{v_0}{(2\pi)^3} \int k^2 dk \sin\theta \, d\theta \, d\varphi \ , \tag{3.2}$$

where the polar axis is in z direction. The normal metal of volume v_0 on the left has the energy spectrum

$$E(\vec{k}) = k^2/2m - \mu + V \ . \tag{3.3}$$

Therefore we have

$$k^2 dk = m[2m(E + \mu - V)]^{1/2} dE \tag{3.4}$$

and

$$\kappa a_B \approx \alpha + \beta(1 - \cos\theta) \ , \tag{3.5}$$

with

$$\alpha \equiv a_B[2m(U_0 - E - \mu)]^{1/2}, \qquad \beta \equiv a_B[2m/(U_0 - E - \mu)]^{1/2}(E + \mu - V) \ . \tag{3.6}$$

For a barrier thickness of $a_B \approx 20$ Å, $2\beta \approx 20$ is a typical value.[10]

Performing the integral over E' and using Eqs. (3.3)–(3.6) we obtain

$$w = (v_0/4\pi L L_0) \int dE \, (E + \mu - V) k_F \, e^{-2\alpha(E)} C(d, E) \times [f(E - V) - f(E)] \times \int_0^1 \rho_{r\perp}(E, \theta) e^{-2\beta(1 - \cos\theta)} d\cos^2\theta \ . \tag{3.7}$$

We have approximated k_z' by k_F because we are only interested in states near the Fermi surface among which only those have an appreciable tunneling probability whose momentum perpendicular to the insulating barrier is very close to the Fermi momentum k_F.

The one-dimensional density of states $\rho_{r\perp}$ is

$$\rho_{r\perp}(E, \theta) = \sum_n \sum_{p=1}^{M} \delta(E - E_{n,p}(\theta)) \ , \tag{3.8}$$

where $E_{n,p}(\theta)$ is a solution of Eqs. (2.1); it is the energy associated with the wave vectors $k_z = n2\pi/2D + p2\pi/L$ and $k_\parallel = k_F \sin\theta$ in the nth band.

Because of the large value of 2β only values of θ very close to zero contribute to (3.7) and

$$d\cos^2\theta \approx 2d\cos\theta \ . \tag{3.9}$$

In Eq. (3.7) we change the integration over $\cos\theta$ into one over $E_{n,p}(\theta)$ and obtain with (3.8) and (3.9)

$$\begin{aligned} I(\epsilon) &\equiv \int_0^1 \rho_{r\perp}(E, \theta) e^{-2\beta(1-\cos\theta)} d\cos^2\theta \\ &\approx 2 \int dE_{n,p}(\theta) \left(\frac{d\cos\theta}{dE_{n,p}}\right) \rho_{r\perp}(E, \theta) e^{-2\beta(1-\cos\theta)} \\ &= 2 \sum_n \sum_{p=1}^{M} \left(\frac{d\cos\theta}{dE}\right) e^{-2\beta[1-\cos\theta(E)]} \ . \end{aligned} \tag{3.10}$$

For $E < \Delta$, the sum in Eq. (3.10) can be evaluated. Equation (2.6) gives

$$\cos\theta(E) = a\epsilon / [\eta(\epsilon) + n\pi] \ ,$$

so that

$$\frac{d\cos\theta}{dE} = \frac{\cos\theta}{E} + \frac{\cos^2\theta}{aE(1 - E^2/\Delta^2)^{1/2}} \ . \tag{3.11}$$

For $\theta \ll 1$, we may approximate

$$\cos\theta \approx 2 - 1/\cos\theta = 2 - [\eta(\epsilon) + n\pi]/a\epsilon . \tag{3.12}$$

The sum over p yields the degeneracy factor M, and we obtain

$$I(\epsilon) = \frac{2M}{\epsilon\Delta} \sum_{n=N}^{\infty} \left(2 - \frac{1}{a\epsilon}(\eta + n\pi) + \frac{2 - [1/(a\epsilon)^2](\eta + n\pi)^2}{a(1 - \epsilon^2)^{1/2}}\right) \times \exp\left[-2\beta\left(\frac{\eta + n\pi}{a\epsilon} - 1\right)\right] \ . \tag{3.13}$$

N is the smallest integer above $[a\epsilon - \eta(\epsilon)]/\pi$ so that for $n \geq N$, $1/\cos\theta - 1 \geq 0$. The large values of n, for which the approximation (3.12) is not valid, contribute negligibly to the sum (3.13). This expression consists of a geometric series of sum S and its first and second derivatives with respect to β;

$$S = e^{(-2\beta/a\epsilon)(\eta + N\pi)} / (1 - e^{(-2\beta/a\epsilon)\pi}) \ . \tag{3.14}$$

For a given ϵ there exists a value $\epsilon_N \geq \epsilon$ such that

$$N = [a\epsilon_N - \eta(\epsilon_N)]/\pi \ . \tag{3.15}$$

This is Eq. (2.6) for $\theta = 0$. Because of the magnitude of a, the approximation $\eta(\epsilon_N) \equiv \arccos\epsilon_N \approx \frac{1}{2}\pi - \epsilon_N$ gives the eigenvalues

$$\epsilon_N \approx (N + \tfrac{1}{2})\pi/(a + 1) \ , \tag{3.16}$$

with a tolerable error except for $\epsilon_N \approx 1$. Thus, the exponent in S is

$$2\beta\,\frac{\eta(\epsilon)+N\pi}{a\epsilon}=2\beta\,\frac{\eta(\epsilon)-\eta(\epsilon_N)+a\epsilon_N}{a\epsilon}\approx 2\beta\left(r_N\,\frac{\delta\epsilon}{\epsilon}+1\right), \tag{3.17}$$

where

$$\delta\epsilon\equiv\epsilon_N-\epsilon,\quad r_N\equiv 1+1/a\,(1-\epsilon_N^2)^{1/2} \tag{3.18}$$

for $\epsilon_{N-1}<\epsilon\le\epsilon_N$. We obtain as the final result

$$I(\epsilon<1)=\frac{2M}{\epsilon\Delta}\,\frac{e^{-2\beta r_N\delta\epsilon/\epsilon}}{1-e^{-2\beta\pi/a\epsilon}}\left\{1+\frac{1}{a(1-\epsilon^2)^{1/2}}-\frac{\pi}{a\epsilon(1-e^{-2\beta\pi/a\epsilon})}\left[1+\frac{2}{a(1-\epsilon^2)^{1/2}}\left(1+\frac{\pi}{a\epsilon(1-e^{-2\beta\pi/a\epsilon})}\right)\right]\right.$$
$$\left.-\,r_N\,\frac{\delta\epsilon}{\epsilon}\left[1+\frac{1}{a(1-\epsilon^2)^{1/2}}\left(2+r_N\,\frac{\delta\epsilon}{\epsilon}+\frac{2\pi}{a\epsilon(1-e^{-2\beta\pi/a\epsilon})}\right)\right]\right\}. \tag{3.19}$$

There is no simple analytic way of calculating $I(\epsilon>1)$, but one may reason that for the lowest states $\Delta<E<2\Delta$, $I(\epsilon>1)$ should behave qualitatively like $I(\epsilon<1)$. The most important term in Eq. (3.19) is $e^{-2\beta r_N\delta\epsilon/\epsilon}$ which produces sharp peaks for tunneling into the levels (bands) with $\vec{k}_\parallel'=\vec{k}_\parallel=0$, because $\delta\epsilon=0$ for $\cos\theta=1$. Since the states with energies $1<\epsilon<2$ are expected still to form very narrow well-separated bands for $\vec{k}_\parallel'=0$, the probability of tunneling into them should vary similarly to $I(\epsilon<1)$.

The net tunneling current from the normal metal into the intermediate state is

$$j_{n\to s}=(e/A)\,w\,,$$

where A is the area of the interface between the two samples. The differential tunneling conductance is

$$e\,\frac{d}{dV}\,j_{n\to s}=\frac{e^2}{A}\,\frac{d}{dV}\,w\,.$$

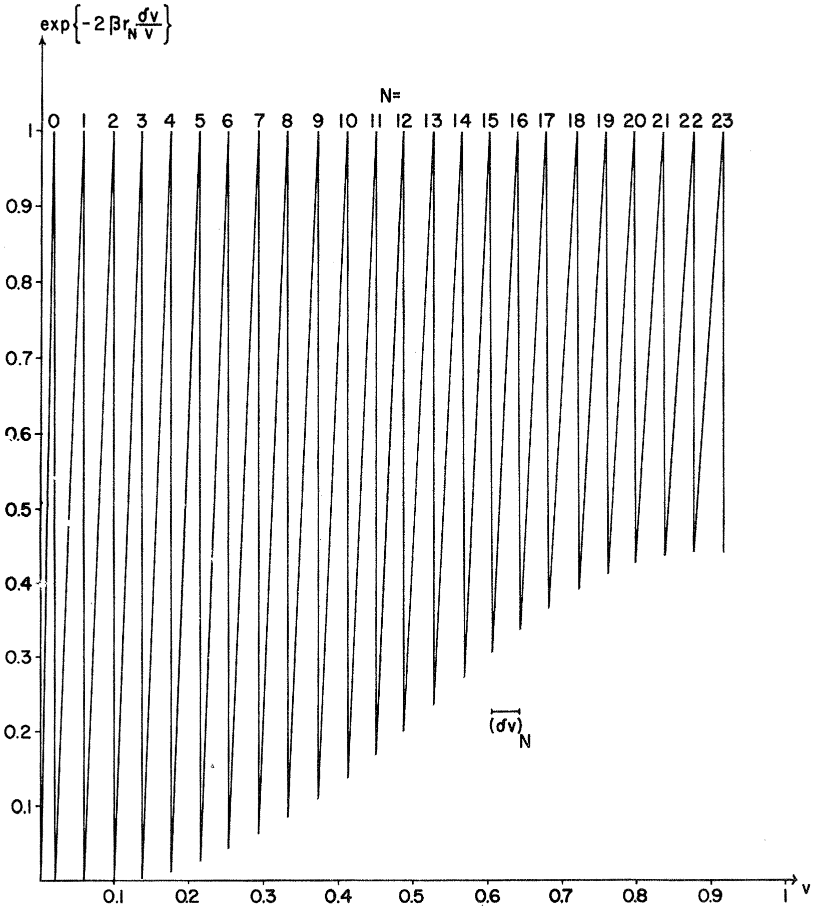

FIG. 2. Leading term $e^{-2\beta r_N\delta v/v}$ of the differential tunneling conductance of Eq. (3.20) vs the reduced bias voltage $v=V/\Delta$ at $T=0°K$, $\beta=20$, and $a=80$; $(\delta v)_N$ is given by Eq. (4.1).

At very low temperatures, $T \approx 0$, the Fermi function $f(E-V)$ in Eq. (3.7) may be treated as a step function with the δ function $\delta(E-V)$ as its derivative. Then, integration over E in Eq. (3.7) yields

$$e\frac{d}{dV}j_{n-s}=\frac{k_F\,\mu}{4\pi L}C(d,V)\,e^{-2\alpha(V)}I(v),\quad v\equiv\frac{V}{\Delta}\ . \tag{3.20}$$

Figure 2 shows the variation of the leading term $e^{-2\beta r_N \delta v/v}$ of $I(v<1)$ of Eq. (3.19) as a function of the reduced bias voltage v.

IV. DISCUSSION

The well-pronounced rapid oscillations of the differential tunneling conductance for $V/\Delta<1$, as shown in Fig. 2, are expected to continue into the range $\Delta<V<2\Delta$. They reflect the band structure in the energy spectrum of the quasiparticles moving normal to the phase boundaries. Finite temperatures will soften the curve and round off the peaks.

The laminar periodic structure of the intermediate state responsible for the effect can be produced by a magnetic field H perpendicular to a thin superconducting slab with a demagnetizing factor $D^*=1$. Alternatively, one may also apply a slanting field to a disk specimen.[5,11] The distance in volts $\delta V_N/e$ between two oscillation peaks is given by Eq. (3.16) with ϵ_N being replaced by $v_N \equiv V_N/\Delta$,

$$\begin{aligned} v_N &\approx (N+\tfrac{1}{2})\pi/(a+1),\\ (\delta v)_N &\equiv v_{N+1}-v_N \approx \pi/(a+1)\approx \pi^2\xi H_c/2DH\ . \end{aligned} \tag{4.1}$$

Compared to the width of the normal regions, $2a_N = 2DH/H_c$, the periodicity $2D$ varies little with the magnetic field. It is always of the order of 10^{-1} cm whereas the coherence length is $\xi \approx 10^{-4}$ cm. Thus, for magnetic fields $H\approx 0.1H_c$ one has $10^{-1} \gtrsim (\delta v)_N \gtrsim 10^{-2}$ so that the voltage difference between the Nth and the $(N+1)$th peak is of the order of 10^{-5} to 10^{-6} V ($\Delta \approx 10^{-4}$ eV).

A complication arises for the states with $E<\Delta$ because of the factor $C(d,V)$. It will heavily damp the tunneling current for $V<\Delta$, if one cannot bring a normal region within a distance $d\approx\xi$ from the tunneling barrier. To facilitate this one could increase the magnetic field so that the width of the superconducting regions decreases. As an unwanted consequence the oscillation periods $(\delta v)_N$ would decrease, because a grows.

However, the smallness of $(\delta v)_N$ in itself should not pose any experimental difficulty, since one now has achieved sensitivities of 10^{-15} V in voltage measurements.[12] The main problem in making visible the band structure of the intermediate state will lie in the temperature smearing of the oscillations of the tunneling conductance. This may require measurements of the second derivative d^2V/dj^2.

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